Aristotle: 384 – 322 BCE

Introduction to Syllogistic Logic; the logic of sets.
Two basic notations

- **Standard form Statements.**
  - All Tigers are felines.
  - No Cats are Dogs
  - Some cats are friendly
  - Some dogs are not friendly.

- **Venn diagrams.**

Either form of notation sufficient for proofs of validity or invalidity.
Aristotle’s Standard Forms

• Goal: standardize language so to reduce confusion and fallacy.

• All expressible statements reduced to “Categorical Statements”.
  – Describe relationships between classes.
  – Only Allowable Verb “to be” (is, are, were)
  – Quantities: All, some (at least one) or none.
  – Statements can be affirmative or negative
  – Nothing else.
Venn Diagrams

• Invented by John Venn c. 1880
• Illustrates the logic of set relations.
Any two sets can have only 3 possible relationships

• One set is a subset of another.

• The sets have nothing in common.

• There is partial overlap between sets.
Venn Diagrams and Categorical Logic

• Standard Drawing:

By Shading or marking the Drawing we can represent any of the set relations expressible in Aristotle’s syllogistic logic.
The standard form statements.

• A form:

• All Subjects are Predicates
  – Ex: All apples are healthy.
  – Quantity: Universal: refers to every thing in its subject class. (it is about every apple)
  – Quality: Affirmative: it makes a positive claim that the subject class is a member of the predicate class.
Venn Diagram for A form

All Dogs are Mammals.

“scribble” marks this area of diagram as EMPTY.
Standard form statements cont.

• E form: No S’s are P’s.
  – Ex: No Apples are poison.
  – Quantity: Universal: Statement refers to every member of subject class.
  – Quality: Negative: Statement denies that subject class is a subset of predicate class.
Venn Diagram for E form

No Bees are Mammals.

“Scribble” marks this area of the diagram as EMPTY.
Standard form statements cont.

• I form: Some S’s are P’s
  – Ex: Some students are hardworking
  – Quantity: Particular: refers to only some of the members of the subject class.
  – Quality: Affirmative: affirms that there is overlap between subject class and predicate class.
  – Interpretation of Some = At least one.
Venn Diagram for I form

Some quadrupeds are Mammals.

“X” marks that at least one set member exists in this region.
Standard Form Sentences (last)

• O Form: Some S are Not P
  – Ex: Some students are not coffee addicts.
  – Quantity: Particular, refers to some, but not all of the subject class
  – Quality: Negative: denies overlap between subject and predicate classes.
Some Dogs are not well trained.

“X” marks that at least one set member exists in this region.
A word about “some”

• In logic, some means “at least one”.

• Notice this implies existence.

• Ancient Greeks like Aristotle didn’t worry about the concept of empty sets.

• This will cause us some grief...
Translations to Standard Form

• No one is interested in philosophy.
  – No people are people interested in philosophy.
    • E form
• Someone is going to win the prize.
  – Some person is a person who wins the prize.
  – More concise: Some person is a prizewinner.
    • I form
• Not every poem is a great work of art.
  – Some poems are not great works of art.
    • O form
• Everyone is going to the bar.
  – All people are people who are going to the bar.
    • A form
Translation Exercises

• Everyone thinks you should study more.
• Alligators are dangerous.
• Don’t play with electricity.
• Many philosophers are taxi drivers.
• A few philosophers are beekeepers.
• Bees don’t sting themselves.
• Not every dog chases its tail.
Some problems with translations

• Reference by name: (direct reference)
  – Ted is going to the lecture.
  – Wrong: Some person is going to the lecture.
  – Right: All people named Ted are going to the lecture.
Translating “only”

• Only is highly contextual.
  Only American Citizens can buy land in Minnesota:
  Wrong: All AC are Minnesota land buyers.
  Right: All Minnesota Land buyers are AC’s.

  Compared to:
  Dogs are the only animals that fetch.
    All fetchers are Dogs.
  The only good car is a Toyota.
    All good cars are Toyotas.
Tough translations

• Only a few students will get A+.
• Only staff can use the staff men’s washroom.
• The platypus is the only mammal with a beak.

• The pope is in favour of peace on earth.
• Queen Elizabeth II visited Winnipeg.
• Justin Bieber’s show sold out.
End Translations and basic Venns

• Do exercises page 105 and 109.
Direct Inference

• Logical relationships exist between any standard form statement and the other 3 possible statement forms using the same subject and predicate.

• Without any argument, you can make inferences about the true/falsity of related statements.
Contradiction.

- Contradiction
  - In Syllogistic Logic: 2 Statements are contradictory when their truth values are always opposite.
  - That is, they cannot both be true and they cannot both be false.
  - The contradiction of an A form statement is an O form statement (and *vice versa*).
  - The contradiction of an E form Statement is an I form statement (and *vice versa*).
Contradictions

• A and O
  – All S are P
  – Some S are not P

• E and I
  – No S are P
  – Some S are P
Examples of Contradiction

• No Dogs are Cute (E form) is contradicted by
  – Some dogs are cute (I form).
  – So if it is true that “no dogs are cute”,
    It directly implies that “some dogs are cute” is false.

• All Dogs are Cute (A form) is contradicted by
  – Some dogs are not cute (O form).
  – So if it is true that “All dogs are cute” then it is false that “some dogs are not cute”.

Contrary

• Two statements are contrary if they cannot both be true at the same time but they can both be false.
  – The contrary of an A form is E (and vice versa).

Example: All leopards are spotted (A form)
  Contrary is No leopards are spotted (E form)
Contradiction vs. Contrary

• People regularly confuse these two concepts.
• Increases the standard of evidence.
• Consider: “All rodents are vermin.”
  – Which is easier to prove?
    No Rodents are Vermin
    Some rodents are not vermin.
Contradiction vs. Contrary

- People regularly confuse these two concepts.
- Increases the standard of evidence.
- Consider: “All rodents are vermin.”
  - The particular statement is much easier!
    - No Rodents are Vermin
    - Some rodents are not vermin.

Contradiction vs. Contrary
Sub-contrary

• Two Statements are sub-contraries if they cannot both be false at the same time, but can both be true.
  – The sub-contrary of an I form is O

Example: Some students are honour-role students
sub-contrary:
Some students are not honour-role students.
Sub-wha?

• To fully understand the concept of sub-contraries: Avoid confusing what is “possible” with “certain”.

If it is true that
   “Some Students are Probationary”
It is possible, but not certain, that
   “Some Students are not Probationary”

Remember, you have no basis in logic for assuming the number of members in a set.
Oh!

• Avoid confusing what is “possible” with what is justified...

If it is **FALSE** that
  “Some Students are Probationary”
It is possible, **AND** certain, that
  “Some Students are not Probationary”

So long as at least one student exists...
Alternation

- A logical relationship also exists between Universal and particular statements of the same quality.
- A’s and I’s or E’s and O’s.
- If the universal is true, the particular must also be true.

Eg: if it is true that “All fish are Slimy”.
It must be true that “Some fish are Slimy”
Sub-alternation

• A logical relationship also exists between Universal and particular statements of the same quality.
• A’s and I’s or E’s and O’s.
• If the particular is false, the universal must also be false.
• eg: if it is false that 
  “Some Dogs are not Smart”
• Then it must be false that 
  “All dogs are not smart”
Traditional Square of opposition

• These logical Relationships can be summarized in a diagram called the “square of opposition”.

![Diagram of the Traditional Square of opposition]
Direct inference exercises.

• What is the contradiction, contrary and sub-alternate of:
  – All Students are hardworking.
  – No Faculty are present.

• What is the contradiction, contrary and alternate of:
  – Some fiddlers are great.
  – Some violinists are not great.

Do exercises page 121.
WARNING: Empty Sets.

• All the direct inference relationships presume:
  – All sets have at least one member.

  Eg: All the present Kings of France are Bald.
  Implies the alternate:
   Some present kings of France are Bald.
  And implies the contradiction:
   Some present kings of France are not Bald.

If you reject this assumption, the only logical relationship that still works is contradiction.
How Aristotle handles empty sets.

• Traditional or Aristotelian Interpretation.
  – No empty sets.
  – All statements that have unfulfilled existential import are false.
    • Eg: All Dragons are Greedy.
      – This statement is interpreted as:
        All dragons, and there are at least one, are greedy.
        So it is obviously false.
How Boole handles empty sets.

• Modern or Boolean Interpretation.
  – Empty sets OK.
  – All statements that have unfulfilled existential import are true.
    • Eg: All Dragons are Greedy.
      – This statement is interpreted as:
        All dragons, if any, are greedy.
        So it is true.
Ordinary Language and Empty Sets

• In ordinary language we usually rely on context to indicate which interpretation we are using:
  – All trespassers are prosecuted.
    • Hopefully an Empty Set.
    • This is still true even if there are no trespassers.
  – Some students are going to pass the test.
    • False if there are no students.
The modern Square of opposition

- All S are P
- No S are P
- Some S are P
- Some S are not P

“Boolean Interpretation”.
Why does “contrary” disappear?

• The contrary of “All S are P” is “No S are P”.
• Traditional interpretation (no empty sets).
  – All Sea Monsters are Dangerous -Both False
  – No Sea Monsters are Dangerous (ex. Import)

• Modern Interpretation (allows empty sets).
  – All Sea Monsters are Dangerous -Both Presumed
  – No Sea Monsters are Dangerous true.

Contraries =defn= cannot both be true.
End: Direct Inferences.

• We omit chapter 3, section 3.8.
Distribution of Terms.

• A term is distributed if the statement it is in refers to every member of the class denoted by the term.
• Otherwise it is not distributed.
• Distributions marked with a flat line over distributed terms, and a U over undistributed terms.
A form distributions

All generals are fools.

• Subject Distributed, predicate undistributed.

Tip: Say to yourself:
Does this refer to All ______, or only some.
E form Distributions

• No Trucks are good trucks
  – Both subject and predicate distributed

This refers to both all trucks and all good trucks.
No truck is a good truck and
No good truck is a truck.
I form distributions

- Some Fish are stinky.
  - No terms are distributed.

Some fish are stinky and some stinky things are fish.
O Form distributions

• Some fish are not edible.
  – Subject undistributed, Predicate distributed.

This one is not obvious:
Clearly fish is not distributed, but the predicate “edible” is distributed because its reference is exclusionary:

Think of this statement as:
Some fish are not in the entire class of edible things.
Distribution summary

• Subjects of Universal statements distributed.
• Predicates of Negative statements distributed.
• All other terms undistributed.

• Distribution exercises page 112.

Summarize everything on master square of opp.
End of Copi’s Chapter 3

• We omitted section 3.8.
• (conversion – obversion – contraposition)
• Interesting but not particularly useful.
Categorical Syllogisms

• Combine 3 standard form statements to form standard form arguments called Standard Categorical Syllogisms (SCS)

• 2 Premises 1 conclusion.

• Question: Can all arguments be reduced to SCS?
  – Sorites: Chains of SCS that combine to form extended arguments.
Aristotle's Theory of the Categorical Syllogism

• A Standard Categorical Syllogism (SCS) is an argument with the following formal features:
  1. It has exactly two premises (and, of course, one conclusion). Each of the premises, and the conclusion, will be a proposition expressed in one of the four standard forms (A, E, I or O).
  2. It contains exactly three terms, each of which occurs twice in the argument.
Names of terms.

• To ease reference, each of the three terms is known by where it occurs in the argument.
  – Minor
  – Major
  – Middle

Depending on the argument, the location of the terms can vary from subject to predicate.
Minor term

- The **minor term** is the one which occurs as the *subject* term in the *conclusion*. It also occurs (as either subject or predicate term) in one of the premises.
- The premise containing the minor term is called the **minor premise**.
Major Term

- The major term is the one which occurs as the *predicate* term in the conclusion. It likewise occurs once, as either subject or predicate term, in one of the premises (but not in the minor premise).

- The premise containing the major term is called the major premise.
Middle Term

--That leaves one term which occurs once in each premise, as either subject or predicate, but not in the conclusion. It is known as the middle term.
Examples

• No Cats (minor) are Dogs (middle).
• All dogs (middle) are Mammals (major).
• No Cats (minor) are Mammals (major).

• All apples (middle) are fruit (minor).
• All Apples (middle) are round (major).
• All fruits (minor) are round (major).
Rules for validity of Syllogisms

• When an argument is expressed as an SCS, a simple set of rules may be applied to determine whether it is deductively valid.
5 Rules (Aristotle’s Rules, no empty sets)
6 Rules (modern interpretation, empty sets ok)
Rule #1

Avoid 4 terms.

The middle term must be exactly the same in each premise. This guarantees relevance.

Failure is called the **Four Term Fallacy**.

Similar to the informal fallacy equivocation.
Counter example 4 term fallacy

• Nothing is better then Steak.
• Spam is better then nothing.
• Nothing is better then steak.

• When “nothing” is recognised as equivocal: and treated as two separate terms, then this fallacy is obviously fallacious.
Rule #2

The middle term must be distributed at least once.

When an argument fails to conform to this rule, it is said to commit the fallacy of the undistributed middle.
Counter Example undistributed Middle

- All dogs are Mammals
- All Cats are mammals
- All Dogs are cats.
Rule #3.

• A term which is distributed in the conclusion must also be distributed in the premise in which it occurs.

--An argument which fails to conform to this rule commits the fallacy known as Illicit Process of the Major Term, or Illicit Process of the Minor Term, as the case may be. The fallacies are called **illicit major** and **illicit minor**, for short.
Counter Example Illicit Minor

- All Conservatives are wealthy.
- All wealthy people are wise.
- All wise people are conservatives.

An unwise conservative is not a counter-example!

A wise non-conservative!
Rule #4

• Avoid 2 negative Premises.

  – Failure = fallacy of exclusive premises.

  – Ex:

    No Cats are Dogs
    No Felines are Dogs
    No Cats are Felines.
Counter Example Exclusive premise

• No alligators are furry.
• Some mammals are not furry.
• Some alligators are mammals.
Rule #5

• If either premise is negative the conclusion must be negative.

• Failure = fallacy of drawing an affirmative conclusion from a negative premise.

  No Cats are Dogs
  All Dogs are Furry
  No Cats are furry.
Drawing a positive conclusion

• Some annoying dogs are small.
• Some large dogs are not annoying dogs.
• Some large dogs are small dogs.
Phil’s rule

• Note that Copi’s rules 4 and 5 can be combined into the rule.

• Number of negative Premises must equal number of negative conclusions.
  – Impossible to have 2 negative conclusions.
  – Requires 1 negative premise for a negative conclusion.
Existential Import

• Remember that Some = “at least one”.

• This causes some odd results.
  All unicorns have horns.
  All horned creatures are dangerous.
  Therefore Some unicorns are Dangerous.

But Some unicorns are dangerous implies at least one unicorn exists!
The existential fallacy in action

• All Gods are things that care about everyone.
• All things that care about everyone are things that care about me.
• Therefore, Some gods are things that care about me.
• Therefore, At least one god exists.
Rule 6 (Boole’s Rule)

- Particular conclusion requires at least one particular premise.
  
  - Failure = existential fallacy.

  - See Copi page 113 for Existential import.

George Boole:
1815 - 1864
Applying the rules.

• Put the argument into SCS form
• Identify the minor, major and middle terms.
• Identify the distribution of terms.
• Apply the rules.
Rule Summary

• R1: Avoid 4 terms.
• R2: Middle term distributed at least once
• R3: Terms distributed in conclusion must be distributed in the premise in which they occur.
• R4: Avoid 2 negative premises.
• R5: Negative conclusion requires 1 negative Premise.
• R6: Particular conclusion requires at least one particular premise.
Venn Diagram Analysis

See Whiteboard.
Problems

• All acts of free will are uncaused events, since all mental decisions are uncaused events and all acts of free will are mental decisions.
More problems

• Many geometric figures aren’t rectangles. Triangles are geometric figures. Thus, rectangles aren’t triangles.

• No persons who plan ahead are persons who live in the present, since all persons who live in the present are satisfied people, and no persons who plan ahead are satisfied people.
More problems

• No thing is better than being successful, but all doing poorly is better than no thing, so all doing poorly is better than being successful.

• Demonstrative knowledge is never immediately certain, and intuitive knowledge is immediate certainty. From this, the conclusion is easily drawn.
More problems 2

• Some snakes are dangerous creatures, for some snakes are poisonous animals and some poisonous animals are dangerous creatures. Valid

• Since no artists are wealthy men and all painters are artists, no painters are wealthy men. Valid
More problems 3

Some children's shows are not violent programs. Some acceptable T.V. shows are not violent programs. Therefore, all children's shows are acceptable T.V. shows.
Invalid: Illicit major & Exclusive Premises.

Sports cars are not very good cars; after all, some sports cars will not start in winter, and any good car will start in winter.
Invalid: Illicit Major & Exclusive Premises.
More problems 4

• Anyone caught stealing cookies from the cookie jar will be punished. It is certain that no one will admit to stealing cookies, hence some of the people punished will not have admitted their guilt.
  – Valid under Aristotle’s rules: Invalid under Boole’s rule; Existential Fallacy.

• Some wasps must be bees, because all bees are yellow, and some wasps are yellow.
  – Invalid, Undistributed Middle.
What fallacy is this?

Penguins are black and white, some old TV shows are black and white, therefore, some penguins are old TV shows.

Logie: another thing that penguins aren’t very good at.
What fallacy is this?
Sorites: Chains of Syllogisms
God is love.
Love is blind.
Therefore God is blind.

God is blind
Ray Charles is blind.
Therefore Ray Charles is God.
Direct reference problem...

• What fallacy does this poor pooch make?

All cats have four legs.
I have four legs.
Therefore, I am a cat.